A METHOD FOR CALIBRATING RESISTANCE THERMOMETRY BRIDGES

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Abstract

A simple, low cost, passive resistance network for calibrating resistance bridges is described and demonstrated. The network, which is closely related to Hamon build-up resistors, is configured so that the four base resistors can be connected to realise 35 distinct four-terminal resistances all interrelated by the usual formulae for the series and parallel connections of resistors. Previous work showed that in the assessment of ac resistance thermometry bridges the network has an accuracy of better than 1 part in 10^8, and for dc bridges an accuracy of 1 part in 10^9. This work demonstrates the application of the network to the calibration of thermometry bridges by including the 35 complement measurements which are obtained by exchanging the bridge connections for the network and the standard resistor. The 70 interrelated results then provide for a full assessment of the accuracy of the bridge. The method is demonstrated by application to the calibration of a 7 digit ac resistance thermometry bridge and a faulty 5 digit bridge. The results of the assessments of 38 different bridges are summarised.

1. INTRODUCTION

Accurate resistance thermometry requires the measurement of the ratio of four-terminal resistances with accuracies exceeding 1 part in 10^6 and approaching 1 part in 10^7. While there are many resistance bridges available for this purpose there remains an almost universal problem: how to calibrate the bridges and demonstrate the accuracy and traceability of both the resistance and temperature measurements.

Currently the accepted methods for calibrating high accuracy bridges [1] rely on disassembly of the bridge and an inter-comparison of the current or voltage divider stages that are the core of the bridge. However these methods are not readily applicable to commercial resistance thermometry bridges because the disassembly requires a working knowledge of the bridge well beyond that reasonably expected of laboratory staff, and may well include proprietary information. Also the bridges are generally not made to facilitate calibration, so the disassembly is not conducive to maintaining a reliable bridge. Overall the techniques are time consuming and prohibitively expensive on a commercial scale.

In practice confidence in the temperature scales realised using resistance bridges is established on the basis of the manufacturers' specifications for the bridges, some simple health checks on the bridges, and evidence of the consistency of scales realised using different bridges. There remain, however, nagging concerns about the real accuracy of the bridges, the ability to recognise faulty bridges, and ultimately the traceability of the temperature scales. As research pushes the accuracy of the scales closer to the accuracy of the bridges and as the internationally accepted norm for traceability tightens, these concerns will become more serious.

This paper demonstrates a novel resistance network [2-4], recently developed by the Measurement Standards Laboratory of New Zealand for the purpose of calibrating thermometry bridges. The network has an accuracy of better than 1 part in 10^8 for ac applications at frequencies typical of thermometry bridges [3] and a dc accuracy approaching 1 part in 10^9 [3,4]. Experiments with the network indicate that the network is fail-safe in the sense that any fault in either the bridge or the network will be manifest as a poor test result [3]. The network is also sufficiently simple that it can be used to make a regular and extensive health checks on a bridge.

The first two sections of the paper outline the principles of the network and describe how it may be used. The final section demonstrates the application of the network to thermometry bridges and summarises the results of the tests on all of the bridges tested to date.

2. PRINCIPLES OF THE NETWORK

The network, as shown in Figure 1 is very similar to a Hamon build-up resistor [5,6]. The main components are the four base resistors, R_1 to R_4, and the four-terminal junction, *J*. The junction, which is engineered to have a four terminal resistance very close to zero, allows the four base resistors to be connected so that they appear to be connected to a single point. This ensures that the electrical definition of the resistances is the same when the resistors are connected singly, in series, or in parallel. A set of switches (not shown in Figure 1) allows the resistors to connected in a total of 35 different series and parallel combinations. The combinations are:

i. any resistor on its own (4 combinations),

ii. any two resistors in series (6 combinations),

iii. any two resistors in parallel (6 combinations),

iv. any two resistors in parallel and in series with a third (12 combinations),

v. any three resistors in parallel and in series with the fourth (4 combinations), and

vi. any two pairs in parallel connected in series (3 combinations).

The additional resistances in the potential and current leads of the four base resistors (R_{P1} to R_{P4} and R_{C1} to R_{C4}) form a combining network that is necessary to ensure the accuracy of combinations that require parallel connections of resistors [3,6].

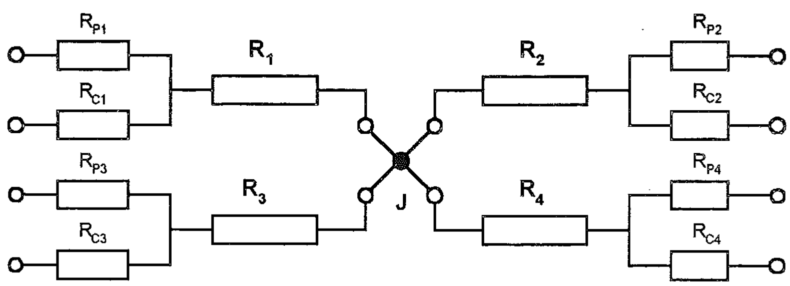


Figure 1: An equivalent circuit for the resistance network. The main components are the base resistors R_1 to R_4, and the four-terminal junction, J. The resistors R_{P1} to R_{P4} and R_{C1} to R_{C4} are the potential and current sharing resistors in the combining network that allow the base resistors to be connected in parallel combinations.

Because the 35 resistances realised with the network are interrelated to the four base resistances by the formulae for series and parallel connections of resistors, the network is completely characterised by four parameters: the base resistances. With the 35 measurements of resistance there is sufficient information on the bridge-network system to calculate values for both the base resistances and the coefficients in an equation describing the errors in the bridge.

All resistance bridges measure resistance by comparing an unknown resistance R_t, with a standard resistance R_s. All 35 measurements of the network resistance are therefore dimensionless ratios interrelated by a single scale factor, the standard resistance R_s. This has two consequences. Firstly because bridges indicate a dimensionless ratio, there is no need for a direct link to the SI base units, and consequently the calibration requirements on the network itself are satisfied very easily. Secondly, the 35 measurements of the network by themselves will provide information only on the linearity of the bridge and not its absolute accuracy. If an absolute calibration is required it is necessary to have a bridge that accommodates an external standard resistor. Then, by exchanging the unknown and standard connections to the bridge, a further 35 'complement' ratios can be obtained. The inclusion of both 'normal' and complement ratios in a bridge assessment ensures that the absolute accuracy is determinable, and provides up to 70 interrelated measurements to assess the accuracy and linearity of the bridge.

The range of resistances realised by the network can be varied by appropriate choice of the four base resistors. For example, a network with two 100 Ω and two 50 Ω resistors generates 12 distinct resistance values that are all simple rational multiples of 100 Ω. With an appropriate choice of base resistors the network will realise 35 distinct resistance values, and ensure that there is a high probability that every numeral of every digit of a bridge reading is exercised [3]. By exercising every digit in this way, the network helps to demonstrate that all of the internal switches and divider stages of the bridge are functioning correctly.

3. USING THE NETWORK

The 70 measurements of the bridge-network system, in effect, include 66 measurements of the behaviour of the bridge. However to accommodate all 70 measurements even with the simplest network (four equal resistors), the bridge must be able to measure ratios greater than 2, so typically a bridge will not accommodate all 70 ratios. Bridges based on the design of Knight [7,8] for example will only accommodate values of standard resistance very close to pre-set values. In these cases only one complement measurement may be possible. For a full calibration it is sufficient that the measurements include a sufficient number of both normal and complement measurements to ensure high confidence in the statistical estimates of the corrections and uncertainties.

With bridges employing only an internal standard resistor, complement measurements are not possible and only the bridge linearity may be determined. For the purposes of resistance thermometry a linearity assessment is usually sufficient because temperature scales are usually defined in terms of resistance ratio and only the linearity of the bridge is important. If required, the absolute accuracy must be determined by comparison with a calibrated resistor.

The results of the measurements made with the network may be analysed in several ways. The simplest is to use the measured values for the ratios P_1 to P_4 (R_1/R_S to R_4/R_S) to calculate the expected readings for the other ratios and compare the results. A more accurate assessment of the accuracy of a bridge is gained by using a least squares fit to find the best values for P_1 to P_4 from all measurements. The least squares technique also provides the means to characterise and model the errors in the bridge.

The analysis proceeds by minimising the variance between the measurements and the predicted values of resistance ratio:

$$s^2 = \frac{1}{N-\rho}\sum_{i=1}^{N}(P_{i,m} + \Delta P(P_{i,m}) - P_i)^2 \qquad (1)$$

where $P_{i,m}$ are the measured ratios, $\Delta P(P_{i,m})$ is a correction equation, and P_i are the values of ratio calculated from the fitted values for the base ratios. Note that the number of degrees of freedom in the variance is equal to the number of measurements N, minus the number of fitted parameters ρ.

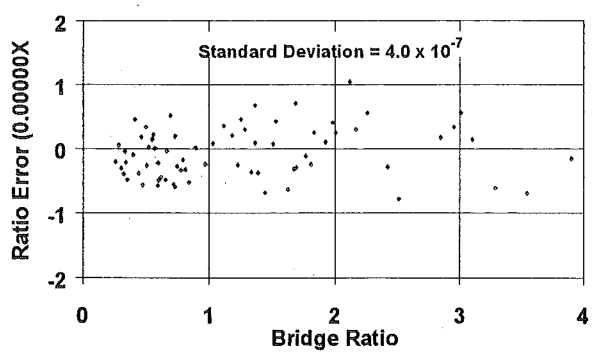
The correction equation is chosen according to the model of the errors that occur in the bridge. A cubic error model which describes an offset, an error of scale, and small even and odd order non-linearities, has been found to be useful. Another possible correction equation is the 'sawtooth' shaped function based on the error associated with the most significant digits of binary and decimal divider stages in bridges. In the simplest analysis the correction is assumed to be zero and only the four base ratios are fitted. If the correction equation accurately describes the errors in the bridge then the standard deviation s given by equation (1) is a measure of the uncertainty in the corrected bridge readings.

The least squares problem defined by equation (1) is non-linear since many of the ratios are non-linear functions of the base ratios. Like all non-linear least squares problems equation (1) is susceptible to multiple minima and a robust least squares algorithm and some care is required to locate the best minimum [9].

4. RESULTS

4.1 Examples of bridge tests.

Figure 2 summarises the results of an assessment of a 7 digit ac bridge with a network covering the range 44 Ω to 390 Ω. With a 100 Ω standard resistor and the inclusion of all 35 complement ratios, the measured ratios cover the range from approximately 0.25 to 3.9. The 70 measurements required for the assessment were taken manually and took approximately 40 minutes. The results show that the bridge is functioning well since the standard deviation of 4.0×10^{-7} is 0.4 of the least significant digit in the reading and is very close to the value of 2.9×10^{-7}, the limit imposed by



quantisation error. Also higher order fits that include possible correction equations fail to improve the quality of the fit indicating that any large scale non-linearity in the bridge is negligible.

Figure 2: The results of a full 70 measurement assessment of a 7 digit ac thermometry bridge. A four parameter least squares fit was first used to calculate the best values for the four base ratios. The error in bridge ratio was then calculated as the difference between the ratio calculated from fitted values and the measured value of that ratio.

Figure 3 shows the results of an analysis of the 35 'normal' measurements for the 7 digit bridge of Figure 2 but with a 50 MΩ resistor shunting the R_t connection to the bridge. The shunting resistor is intended to simulate a fault, perhaps due to poor connecting cables, or a faulty bridge with one stage of a multistage transformer not functioning. With the resistor in place the measured ratio for the bridge is

$$P_m = \frac{1}{R_S}\frac{R_N R_L}{R_N + R_L} \tag{2}$$

$$\approx \frac{R_N}{R_S} - \left(\frac{R_N}{R_S}\right)^2 \frac{R_S}{R_L} \tag{3}$$

where R_N is the network resistance and R_L is the 50 MΩ resistor. Thus with the shunting resistor the bridge should exhibit a small error that has a quadratic dependence on the measured ratio. Figure 3(a) shows the results of a least squares fit to the four base ratios only, and Figure 3(b) shows the fit to the four base ratios plus a quadratic correction term. The presence of the fault is revealed by the significant improvement in the standard deviation and the statistically significant estimate for the coefficient in the correction equation. With the correction included the standard deviation for the fit is the same as that in Figure 2. Also the value of R_L inferred from the fit was 49.5 MΩ ± 1.6 MΩ, which is very close to the expected value.

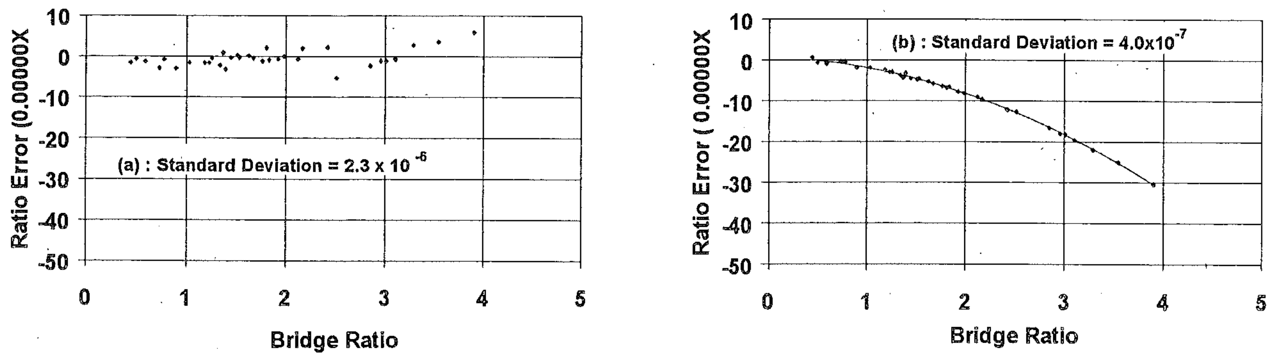


Figure 3: The results of a 35 measurement test on the bridge of Figure 2 with a 50 MΩ resistor shunting the R_t connection to the bridge: (a) the residual error from a four parameter fit to the base ratios only; (b) the residual error from a five parameter fit to the four base ratios and a quadratic correction term (see text). The solid line represents the quadratic correction equation.

Figure 4 shows the results of measurements and analysis for a 5 digit ac bridge that was found to be faulty. Figure 4(a) shows the residuals for the fit to base ratios only and Figure 4(b) shows the fit to base ratios plus a general cubic equation of the form

$$\Delta P = A_0 + A_2 P^2 + A_3 P^3 . \quad (4)$$

Note that a linear term has not been included in equation (4). Because the bridge has an internal standard resistor and only the 35 normal ratios could be measured, only the non-linearity of the bridge is determinable. As with the bridge of Figure 3 the presence of the fault is revealed by significant decrease in the standard deviation with the inclusion of the correction equation in the fit. Also the standard deviation for the fit to base ratios only, was large compared to the manufacturers specification. A test of the bridge after repair showed that the standard deviation for the fit to base ratios dropped to 7×10^{-6}, a value consistent with the manufacturers specification.

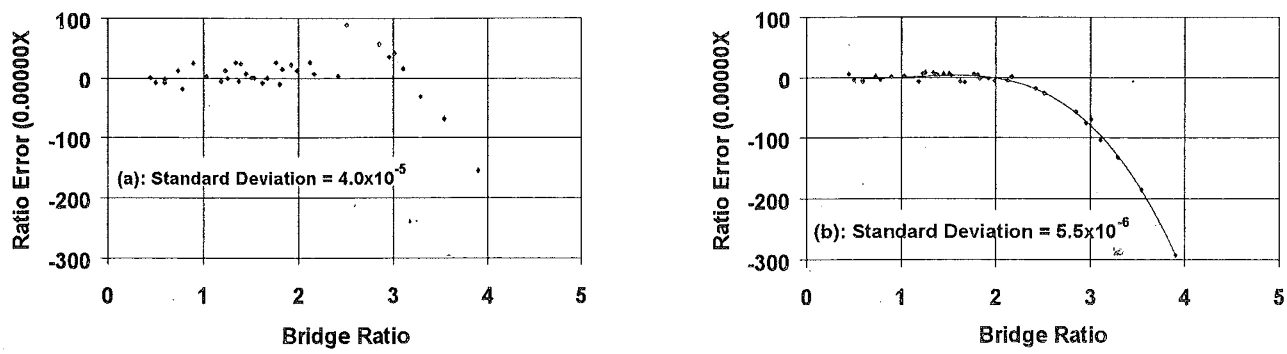


Figure 4 : The results of a 35 measurement test on a 5 digit ac bridge that was found to depart significantly from the manufacturers specifications: (a) the residual error from a four parameter fit to the base ratios only; (b) the residual error from a seven parameter fit to the four base ratios and a three parameter cubic error equation (see text). The solid line represents the cubic correction equation.

4.2 Summary of results to date

To date nearly 100 different tests have been carried out on 38 different ac and dc resistance bridges using several variations of the network. The results of the bridge assessments are summarised in Table 1. Most of the assessments were made using only the 35 'normal' ratios. Complement ratios were included in the assessments of 4 of the bridges.

Bridge resolution	Number of bridges tested	Number of models represented	Number of failures	Comments
> 8 digits	1	1	0	
8 digits	8	2	2	1 suspected to be faulty
7 digits	11	4	2	1 known to be faulty
6 digits	9	2	1	1 suspected to be faulty
5 digits	9	4	3	2 known to be faulty
TOTAL	38	13	8	3 unsuspected faulty

Table 1: A summary of the results of tests of resistance bridges with the network.

The results show that approximately one in five of the bridges tested were found to be faulty. While this is very similar to the failure rate found for other instruments, it is disturbing to find that 3 of the 8 faulty bridges had not been suspected of being faulty. On two occasions the network was used to demonstrate the good health of a bridge previously suspected of being faulty. The problems previously attributed to the bridges were later found to be due to other causes.

The bridge faults uncovered by the network were varied and included a faulty switch, a large offset due to a faulty lead eliminator circuit, excessive noise due to a faulty preamplifier, a faulty transformer, as well as several gross non-linearities similar to that shown in Figure 4. Least squares fits with a sawtooth function proportional to the fractional part of the bridge reading uncovered traces of the sawtooth error in several healthy bridges and one bridge with a significant sawtooth error. Numerical experiments, similar to that on Figure 3, with various models of bridge and network error showed that any errors in either the bridge or the network will be exposed as a poor standard deviation.

It was found that the performance of healthy ac resistance bridges is generally very good. The standard deviations for four parameter fits for all healthy bridges were typically a factor of 3 to 5 below the accuracy specified by the manufacturer. Also fits that included correction equations gave little improvement to the standard deviation, indicating that correction equations are unnecessary for healthy ac bridges. The good quality of the bridges indicates that the design principles used by the manufacturers are sound, and that the standard deviation as defined by equation (1) is a good measure of the accuracy of a bridge.

5. CONCLUSIONS

A wide range of experiments has demonstrated that the network provides the means to detect, and in some cases diagnose, a wide range of faults in resistance bridges. This combined with the simplicity of the network means that thorough and frequent checks on bridges can now be carried out on bridges at low cost. With an appropriate choice of error model, the network and a least squares fit provide the means to determine the corrections to bridge readings and the uncertainty in the readings. Since the accuracy of the network currently exceeds that of the best commercial thermometry bridges it provides the means to calibrate the bridges and hence to demonstrate the traceability of temperature measurements based on resistance thermometry.

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